

# Decoherence by a spin thermal bath: Role of the spin-spin interactions and initial state of the bath

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We study the decoherence of two coupled spins that interact with a spin-bath environment. It is shown that the connectivity and the coupling strength between the spins in the environment are of crucial importance for the decoherence of the central system. For the anisotropic spin-bath, changing the connectivity or coupling strengths changes the decoherence of the central system from Gaussian to exponential decay law. The initial state of the environment is shown to affect the decoherence process in a qualitatively significant manner.

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## I. INTRODUCTION

Understanding the decoherence in quantum spin systems is a subject of numerous works (for reviews, see Refs<sup>1,2</sup>). The issue seems to be very complicated and despite many efforts, even some basic questions about character of the decoherence process are unsolved yet. Due to the interactions with and between the spin of the bath, an analytical treatment can be carried out in exceptional cases, even if the central systems contains one spin only. Recent work suggests that the internal dynamics of the environment can be crucial to the decoherence of the central system<sup>3,4,5,6,7,8,9,10,11,12,13,14,15</sup>. In this paper, we present results of extensive simulation work of a two-spin system interacting with a spin-bath environment and show that the decoherence of the two-spin system can exhibit different behavior, depending on the characteristics of the coupling with the environment, the internal dynamics and the initial state of the latter. We also provide a simple physical picture to understand this behavior.

In general, the behavior of an open quantum system crucially depends on the ratio of typical energy differences of the central system  $\delta E_c$  and the energy  $E_{ce}$  which characterizes the interaction of the central system with the environment. The case  $\delta E_c \ll E_{ce}$  has been studied extensively in relation to the ‘‘Schrödinger cat’’ problem and the physics is quite clear<sup>16,17</sup>: As a result of time evolution, the central system passes to one of the ‘‘pointer states’’<sup>17</sup> which, in this case, are the eigenstates of the interaction Hamiltonian  $H_{ce}$ . The opposite case,  $\delta E_c \gg E_{ce}$  is less well understood. There is a conjecture that in this case the pointer states should be eigenstates of the Hamiltonian  $H_c$  of the central system but this has been proven for a very simple model only<sup>18</sup>. On the other hand, this case is of primary interest if, say, the central system consists of electron spins whereas the environment are nuclear spins, for instance if one considers the possibility of quantum computation using molecular magnets<sup>19,20</sup>.

## II. MODEL

We consider a generic quantum spin model described by the Hamiltonian  $H = H_c + H_{ce} + H_e$  where  $H_c = -\mathbf{J}\mathbf{S}_1 \cdot \mathbf{S}_2$  is the Hamiltonian of the central system and the Hamiltonians of the environment and the interaction of the central system with the environment are given by

$$\begin{aligned} H_e &= - \sum_{i=1}^{N-1} \sum_{j=i+1}^N \sum_{\alpha} \Omega_{i,j}^{(\alpha)} I_i^{\alpha} I_j^{\alpha}, \\ H_{ce} &= - \sum_{i=1}^2 \sum_{j=1}^N \sum_{\alpha} \Delta_{i,j}^{(\alpha)} S_i^{\alpha} I_j^{\alpha}, \end{aligned} \quad (1)$$

respectively. The exchange integrals  $J$  and  $\Omega_{i,j}^{(\alpha)}$  determine the strength of the interaction between spins  $\mathbf{S}_n = (S_n^x, S_n^y, S_n^z)$  of the central system, and the spins  $\mathbf{I}_n = (I_n^x, I_n^y, I_n^z)$  in the environment, respectively. The exchange integrals  $\Delta_{i,j}^{(\alpha)}$  control the interaction of the central system with its environment. In Eq. (1), the sum over  $\alpha$  runs over the  $x$ ,  $y$  and  $z$  components of spin-1/2 operators  $\mathbf{S}$  and  $\mathbf{I}$ . In the sequel, we will use the term ‘‘Heisenberg-like’’  $H_e$  ( $H_{ce}$ ) to indicate that each  $\Omega_{i,j}^{(\alpha)}$  ( $\Delta_{i,j}^{(\alpha)}$ ) is a uniform random number in the range  $[-\Omega, \Omega]$  ( $[-\Delta, \Delta]$ ),  $\Omega$  and  $\Delta$  being free parameters. In earlier work<sup>14,15</sup>, we found that a Heisenberg-like  $H_e$  can induce close to perfect decoherence of the central system and therefore, we will focus on this case only.

The bath is further characterized by the number of environment spins  $K$  with which a spin in the environment interacts. If  $K = 0$ , each spin in the environment interacts with the central system only.  $K = 2$ ,  $K = 4$  or  $K = 6$  correspond to environments in which the spins are placed on a ring, square or triangular lattice, respectively and interact with nearest-neighbors only. If  $K = N - 1$ , each spin in the environment interacts with all the other spins in the environment and, to give this case a name, we will refer to this case as ‘‘spin glass’’.

If the Hamiltonian of the central system  $H_c$  is a pertur-

bation relative to the interaction Hamiltonian  $H_{ce}$ , the pointer states are eigenstates of  $H_{ce}$ <sup>17</sup>. In the opposite case, that is the regime  $|\Delta| \ll |J|$  that we explore in this paper, the pointer states are conjectured to be eigenstates of  $H_c$ <sup>18</sup>. The latter are given by  $|1\rangle \equiv |T_1\rangle = |\uparrow\uparrow\rangle$ ,  $|2\rangle \equiv |S\rangle = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$ ,  $|3\rangle \equiv |T_0\rangle = (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)/\sqrt{2}$ , and  $|4\rangle \equiv |T_{-1}\rangle = |\downarrow\downarrow\rangle$ , satisfying  $H_c|S\rangle = (3J/4)|S\rangle$  and  $H_c|T_i\rangle = (-J/4)|T_i\rangle$  for  $i = -1, 0, 1$ .

The simulation procedure is as follows. We generate a random superposition  $|\phi\rangle$  of all the basis states of the environment. This state corresponds to the equilibrium density matrix of the environment at infinite temperature. Alternatively, to study the effect of the thermal state of the environment on the decoherence processes, we take the 2 state of the environment to be its ground state. The spin-up – spin-down state ( $|\uparrow\downarrow\rangle$ ) is taken as the initial state of the central system. Thus, the initial state of the whole system reads  $|\Psi(t=0)\rangle = |\uparrow\downarrow\rangle |\phi\rangle$  and is a product state of the state of the central system and the initial state of the environment which, in general is a (very complicated) linear combination of the  $2^N$  basis states of the environment. In our simulations we take  $N = 16$  which, from earlier work<sup>14,15</sup>, is sufficiently large for the environment to behave as a “large” system.

For a given, fixed set of model parameters, the time evolution of the whole system is obtained by solving the time-dependent Schrödinger equation for the many-body wave function  $|\Psi(t)\rangle$ , describing the central system plus the environment<sup>21</sup>. It conserves the energy of the whole system to machine precision. We monitor the effects of the decoherence by computing the the matrix elements of the reduced density matrix  $\rho(t)$  of the central system.

As explained earlier, in the regime of interest  $|\Delta| \ll |J|$ , the pointer states are expected to be the eigenstates of the central systems. Hence we compute the matrix elements of the density matrix in the basis of eigenvectors of the central system. We also compute the time dependence of quadratic entropy  $S_c(t) = 1 - \text{Tr} \rho^2(t)$  and the Loschmidt echo  $L(t) = \text{Tr}(\rho(t)\rho_0(t))$ <sup>22</sup>, where  $\rho_0(t)$  is the density matrix for  $H_{ce} = 0$ .

### III. ISOTROPIC COUPLING TO THE BATH

If the interaction between the central system and environment is isotropic we have  $[H_c, H_{ce}] = 0$ . Then, as shown in the Appendix, the expressions of the reduced density matrix  $\rho(t)$  and the Loschmidt echo  $L(t)$  simplify. Indeed, if  $\Delta_{i,j}^{(x)} = \Delta_{i,j}^{(y)} = \Delta_{i,j}^{(z)} \equiv \Delta$  for all  $i, j$ , then

$$H_{ce} = -\Delta(\mathbf{S}_1 + \mathbf{S}_2) \cdot \sum_{j=1}^N \mathbf{I}_j \quad (2)$$

commutes with  $H_c$  and it follows that the decoherence process of the central system is determined by  $H_{ce}$ ,  $H_e$ , the initial state of whole system  $|\Psi(t_0)\rangle$ , and the eigenstates of the central system (see Eq. (15) and (16) in

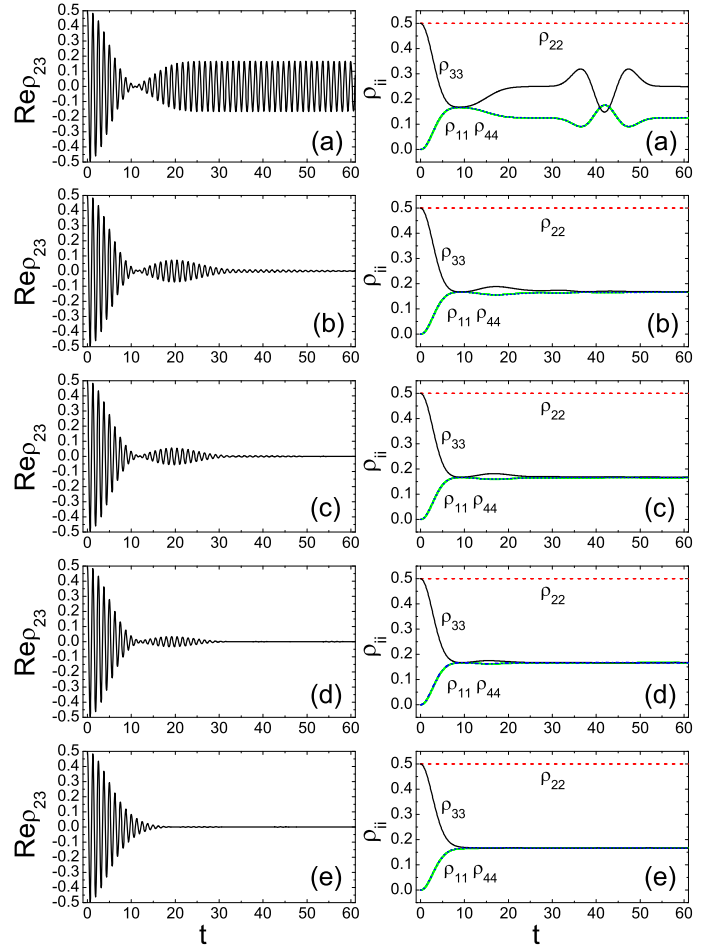


FIG. 1: (Color online) The time evolution of the real part of the off-diagonal element  $\rho_{23}$  (left panel) and the diagonal elements  $\rho_{11}, \dots, \rho_{44}$  (right panel) of the reduced density matrix of a central system (with  $J = -5$ ), coupled via an isotropic Heisenberg interaction  $H_{ce}$  ( $\Delta = -0.075$ ) to a Heisenberg-like environment  $H_e$  ( $\Omega = 0.15$ ) with different connectivity: (a)  $K = 0$ ; (b)  $K = 2$ ; (c)  $K = 4$ ; (d)  $K = 6$ ; (e)  $K = N - 1$ .

the Appendix). In other words, in this case,  $L(t)$  and  $|\rho(t)|$  do not depend on the  $J$ , the interaction between the spins in the central system. Furthermore, if we take the interactions between the environment spins to be isotropic, that is,  $\Omega_{i,j}^{(x)} = \Omega_{i,j}^{(y)} = \Omega_{i,j}^{(z)} \equiv \Omega_{i,j}$  for all  $i, j$ , then

$$H_e = - \sum_{i=1}^{N-1} \sum_{j=i+1}^N \Omega_{i,j} \mathbf{I}_i \cdot \mathbf{I}_j \quad (3)$$

commutes with  $H_{ce}$ , and therefore  $H_e$  has no effect on the decoherence process (see Eq. (18) in the Appendix).

In Fig. 1 and Fig. 2, we show the time evolution of the elements of the reduced density matrix  $\rho(t)$  for different connectivity  $K$  and  $\Omega$ , for the case that  $H_{ce}$  is an isotropic Heisenberg model, i.e.,  $\Delta_{i,j}^{(x)} = \Delta_{i,j}^{(y)} = \Delta_{i,j}^{(z)} \equiv \Delta$  for all  $i, j$ .

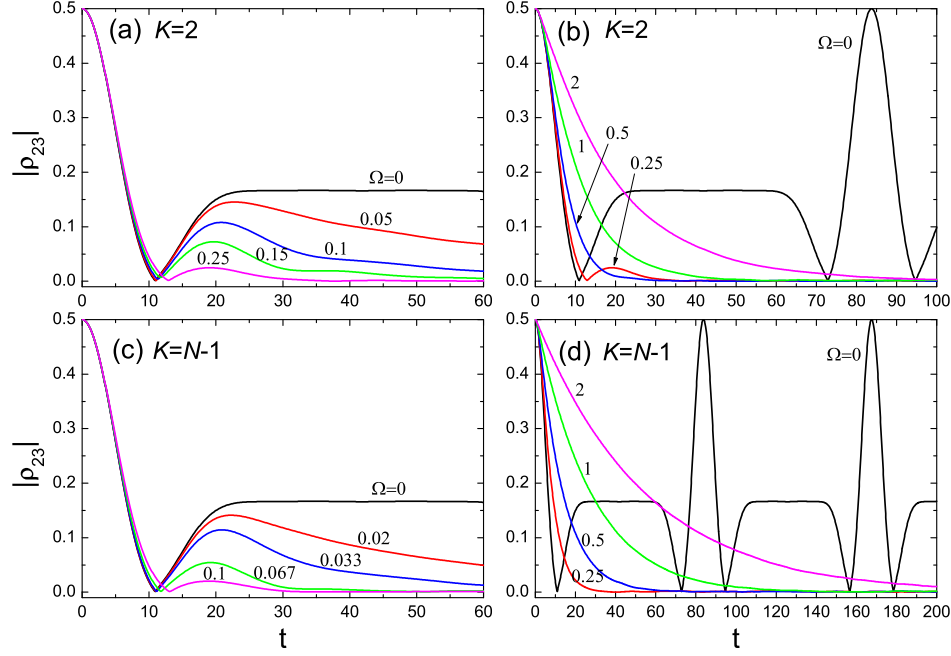


FIG. 2: (Color online) The time evolution of the off-diagonal element  $\rho_{23}$  of the reduced density matrix of a central system (with  $J = -5$ ), interacting with a Heisenberg-like environment  $H_e$  via an isotropic Heisenberg Hamiltonian  $H_{ce}$  (with  $\Delta = -0.075$ ) for the same geometric structures in the environment: (a,b)  $K = 2$  and (c,d)  $K = N - 1$ . The number next to each curve is the corresponding value of  $\Omega$ .

If  $|\Delta| \gg \sqrt{K}\Omega$ , in agreement with earlier work<sup>23,24</sup>, we find that in the absence of interactions between the environment spins ( $\sqrt{K}\Omega = 0$ ) and after the initial decay, the central system exhibits long-time oscillations (see Fig. 1(a)(left)). In this case and in the limit of a large environment, we have<sup>24</sup>

$$\text{Re } \rho_{23}(t) = \left[ \frac{1}{6} + \frac{1 - bt^2}{3} e^{-ct^2} \right] \cos \omega t, \quad (4)$$

where  $b = N\Delta^2/4$ ,  $c = b/2$  and  $\omega = J - \Delta$ . Equation (4) clearly shows the two-step process, that is, after the initial Gaussian decay of the amplitude of the oscillations, the oscillations revive and their amplitude levels off<sup>24</sup>. Due to conservation laws, this behavior does not change if the environment consists of an isotropic Heisenberg system ( $\Omega_{i,j}^{(\alpha)} \equiv \Omega$  for all  $\alpha, i$  and  $j$ ), independent of  $K$ . If, as in Ref.<sup>23</sup>, we take  $\Delta_{i,j}^{(x)} = \Delta_{i,j}^{(y)} = \Delta_{i,j}^{(z)} \in [0, \Delta]$  random instead of the identical, the amplitude of the long-living oscillations is no longer constant but decays very slowly<sup>23</sup> (results not shown).

If  $|\Delta| \approx \sqrt{K}\Omega$ , the presence of Heisenberg-like interactions between the spins of the environment has little effect on the initial Gaussian decay of the central system, but it leads to a reduction and to a decay of the amplitude of the long-living oscillations. The larger  $K$  (see Fig. 1(b-e)(left)) or  $\Omega$  (see Fig. 2(a,c)), the faster the decay is. Note that for the sake of clarity, we have suppressed the fast oscillations by plotting instead of the

real part, the absolute value of the matrix elements.

If  $|\Delta| \ll \sqrt{K}\Omega$ , keeping  $K$  fixed and increasing  $\Omega$  smoothly changes the initial decay from Gaussian (fast) to exponential (slow), and the long-living oscillations are completely suppressed (see Fig. 2(b,d)). For large  $\Omega$ , the simulation data fits very well to

$$|\rho_{23}(t)| = \frac{1}{2} e^{-A_K(\Omega)t}, \quad (5)$$

with  $A_K(\Omega) \approx \Omega \tilde{A}_K$ ,  $\tilde{A}_2 = 9.13$  and  $\tilde{A}_{N-1} = 26.73$ . Note that, in principle, a closed quantum system cannot exhibit exponential decay<sup>25</sup>. The fact that we observe a decay that is well described by a single exponential may be the result of tracing out the degrees of freedom of an environment which initially is in a state of random superposition of the basis states.

Physically, the observed behavior can be understood as follows. If  $|\Delta| \approx \sqrt{K}\Omega$ , a bath spin is affected by roughly the same amount by the motion of both the other bath spins and by the two central spins. Therefore, each bath spin has enough freedom to follow the original dynamics, much as if there were no coupling between bath spins. This explains why the initial Gaussian decay is insensitive to the values of  $K$  or  $\Omega$ . After the initial decay, the whole system is expected to reach a stationary state, but because of the presence of Heisenberg-like interactions between the bath spins, a new stationary state of the bath is established, suppressing the long-living oscillations.

For increasing  $K$ , the distance between two bath spins,

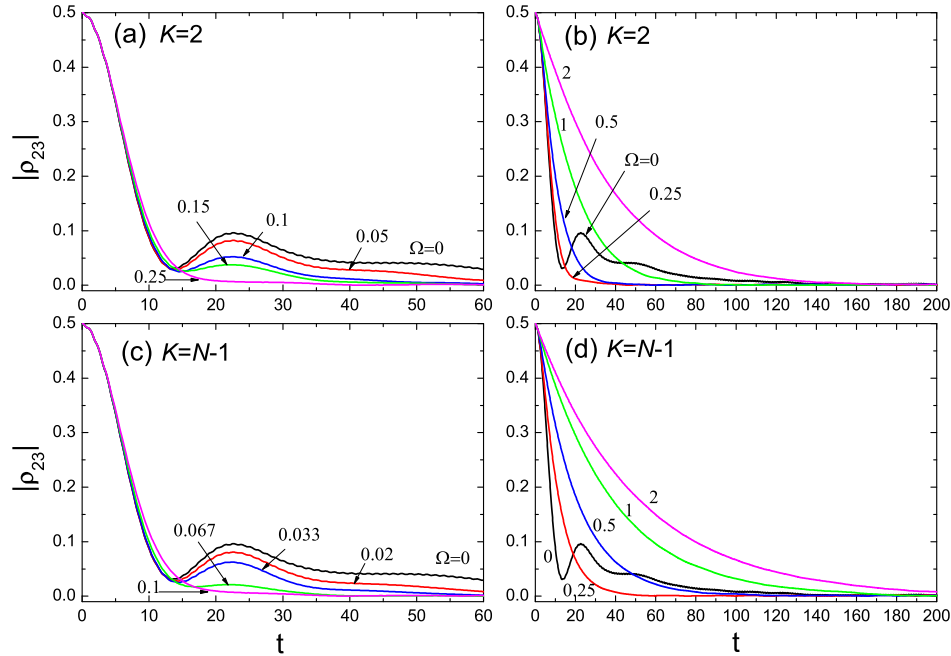


FIG. 3: (Color online) Same as Fig. 2 except that  $H_{ce}$  is Heisenberg-like and  $\Delta = 0.15$ .

defined as the minimum number of bonds connecting the two spins, becomes smaller. For instance, for  $K = 2$ , this distance is  $(N - 2)/2$ , and for  $K = N - 1$ , it is zero. Therefore, for fixed  $\Omega$  and increasing  $K$  the fluctuations in the spin bath can propagate faster and the evolution to the stationary state will be faster. Similarly, for fixed  $K$ , increasing the coupling strength between the bath spins will speed up the dynamics of the bath, that is, the larger  $\Omega$  the faster will be the evolution to the stationary state.

In the opposite case  $|\Delta| \ll \sqrt{K}\Omega$ ,  $H_{ce}$  is a small perturbation relative to  $H_e$  and the coupling between bath spins is the dominant factor in determining the dynamics of the bath spins. Therefore, by increasing  $K$  or  $\Omega$ , the bath spin will have less freedom to follow the dynamics induced by the coupling to the two central spins, the influence of the bath on the central system will decrease, and the (exponential) decay will become slower.

According to the general picture of decoherence<sup>17</sup>, for an environment with nontrivial internal dynamics that initially is in a random superposition of all its eigenstates, we expect that the central system will evolve to a stable mixture of its eigenstates. In other words, the decoherence will cause all the off-diagonal elements of the reduced density matrix to vanish with time. In the case of an isotropic Heisenberg coupling between the central system and the environment,  $H_c$  commutes with the Hamiltonian  $H$ , hence the energy of the central system is a conserved quantity. Therefore, the weight of the singlet  $|S\rangle$  in the mixed state should be a constant (1/2), and the weights of the degenerate eigenstates  $|T_0\rangle$ ,  $|T_{-1}\rangle$  and  $|T_1\rangle$  are expected to become the same (1/6). As shown in Fig. 1(b-e)(right), our simulations confirm that this

picture is correct in all respects.

#### IV. ANISOTROPIC COUPLING TO THE BATH

In order to clarify the role of  $K$  and  $\Omega$ , we change the coupling between the central system and the bath from Heisenberg to Heisenberg-like. From a comparison of the data in Fig. 2 and Fig. 3, it is clear that the roles of  $K$  and  $\Omega$  are the same in both cases, no matter whether the coupling to the bath is isotropic or anisotropic. However, there are some differences in the decoherence process. The most important parameter determining the decoherence process is the ratio of the typical interaction energy  $\Delta$  to the mean-square energy of interactions in the thermal bath,  $\sqrt{K}\Omega$ .

If  $|\Delta| \gg \sqrt{K}\Omega$ , in the presence of anisotropic interactions between the central system and the environment spins, even in the absence of interactions between the bath spins, the second step of the oscillations decays and finally disappears as  $K$  increases. This is because the anisotropic interactions break the rotational symmetry of the coupling between central system and environment which is required for the long-living oscillations to persist.

If  $|\Delta| \ll \sqrt{K}\Omega$ ,  $|\rho_{23}(t)|$  can still be described by Eq. (5), but now  $A_K(\Omega)$  is no longer a linear function of  $\Omega$ . For anisotropic  $H_{ce}$ , the energy of the central system is no longer a conserved quantity. Therefore there will be energy transfer between the central system and the environment and the weight of each pointer state (eigenstate) in the final stable mixture need not be the same

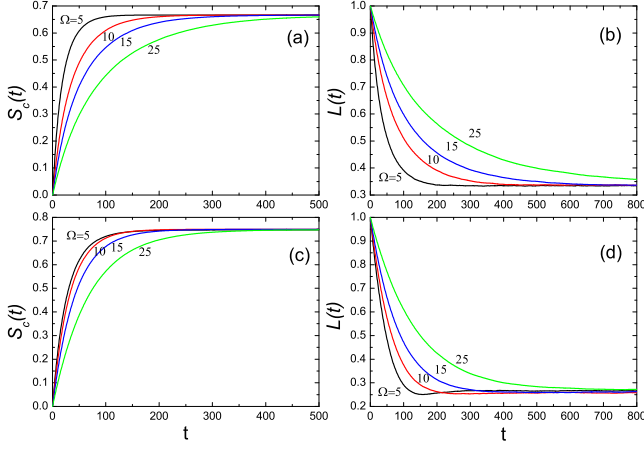


FIG. 4: (Color online) The time evolution of the the entropy  $S_c(t)$  and Loschmidt echo  $L(t)$  of a central system (with  $J = -5$ ), interacting with a Heisenberg-like environment  $H_e$  (with different  $\Omega$ ) via a Heisenberg (a,b,  $\Delta = -0.075$ ) or Heisenberg-like (c,d,  $\Delta = 0.15$ ) Hamiltonian  $H_{ce}$  for the case  $K = 2$ . The number next to each curve is the corresponding value of  $\Omega$ .

for all  $K$  or  $\Omega$ .

For a change, we illustrate this point by considering the quadratic entropy  $S_c(t)$  and Loschmidt echo  $L(t)$ . We expect that these quantities will also dependent of the symmetry of the coupling between central system and the spin bath. In Fig. 4, we present results for large  $\Omega$  and  $K = 2$ , confirming this expectation. For isotropic (Heisenberg)  $H_{ce}$  and perfect decoherence (zero off-diagonal terms in the reduced density matrix) we expect that  $\max_t S_c(t) = 1 - [(1/2)^2 + 3 \times (1/6)^2] = 2/3$ , in concert with the data of Fig. 4(a)). For Heisenberg-like  $H_{ce}$ ,  $\max_t S_c(t)$  will depend on the coupling strengths and as shown in Fig. 4(c), we find that  $\max_t S_c(t) = 1 - 4 \times (1/4)^2 = 3/4$ , corresponding to the case that all the diagonal elements in the reduced density matrix are the same  $(1/4)$  and all other elements are zero.

## V. DISCUSSION AND CONCLUSIONS

In the foregoing, we have compared  $\sqrt{K}\Omega$  to  $|\Delta|$  to distinguish different regimes. As a matter of fact,  $\sqrt{K}\Omega$  does not completely characterize the decoherence process, but it can be used to characterize its time scale. Indeed, as shown in Fig. 5, for different  $\sqrt{K}$  and  $\Omega$  but the same value of  $\sqrt{K}\Omega$ , the the time evolution of  $L(t)$  is very similar. Note that if  $\sqrt{K}\Omega$  increases (compare Fig. 5a to Fig. 5d), the differences between the Loschmidt echoes increase. Additional simulations (results not shown) indicate that this differences are fluctuations that are due to the particular realization random parameters used in the simulation.

In conclusion, for a spin-bath environment that ini-

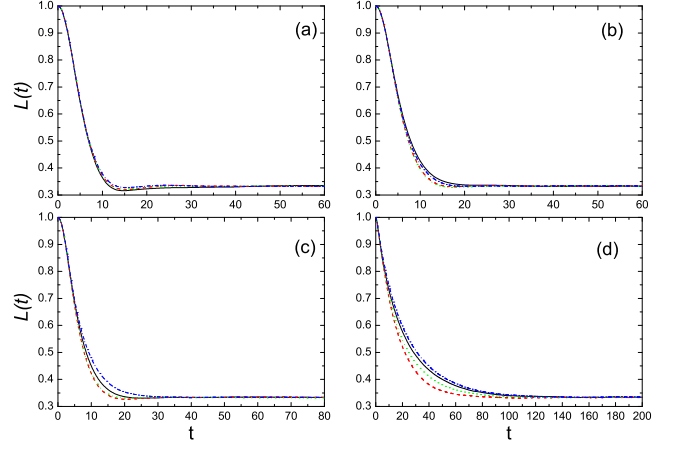


FIG. 5: (Color online) The time evolution of the Loschmidt echo  $L(t)$  of a central system (with  $J = -5$ ), interacting with a Heisenberg-like environment  $H_e$  via a Heisenberg ( $\Delta = -0.075$ ) Hamiltonian  $H_{ce}$ . In each panel, the values of  $\sqrt{K}\Omega$  are the same: (a)  $\sqrt{K}\Omega \equiv 0.1\sqrt{N-1}$ , (b)  $\sqrt{K}\Omega \equiv 0.15\sqrt{N-1}$ , (c)  $\sqrt{K}\Omega \equiv 0.25\sqrt{N-1}$ , and (d)  $\sqrt{K}\Omega \equiv \sqrt{N-1}$ . The different lines in each pannel correspond to different  $K$ . Solid (black) line:  $K = 2$ ; Dashed (red) line:  $K = 4$ ; Dotted (green) line:  $K = 6$ , and dash-dotted (blue) line:  $K = N - 1$ .

tially is in a random superposition of its basis states, we have shown how a pure quantum state of the central spin system evolves into a mixed state, and that if the interaction between the central system and environment is much smaller than the coupling between the spins in the central system, the pointer states are the eigenstates of the central system. Both these observations are in concert with the general picture of decoherence<sup>17</sup>. Furthermore, we have demonstrated that, in the case that the environment is a spin system, the details of this spin system are important for the decoherence of the central system. In particular, we have shown that for the anisotropic spin-bath, changing the internal dynamics of the environment (geometric structure or exchange couplings) may change the decoherence of the central spin system from Gaussian to exponential decay.

Finally, we would like to compare the present results with those of our earlier work in which we focussed on the case in which the environment is initially in its ground state and demonstrated that, apart from the strength of different interactions, also their symmetry and the amount of entanglement of the ground state of the central system affects the decoherence<sup>14,15</sup>. To facilitate the comparison, in Fig. 6 we present some new data of the Loschmidt echoes for different  $K$  but for fixed  $\sqrt{K}\Omega$ . Comparison of Fig. 5 with Fig. 6 indicates that if the environment is initially in its ground state, the decoherence process is qualitatively different from the one observed in the case that the initial state of the environment is a random superposition. Roughly speaking, it

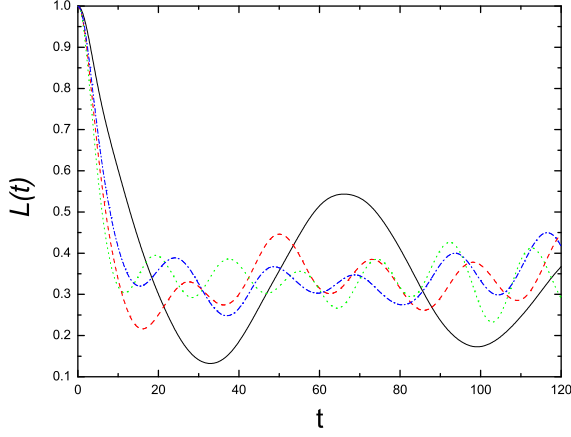


FIG. 6: (Color online) The time evolution of the Loschmidt echo  $L(t)$  of a central system (with  $J = -5$ ), interacting with a Heisenberg-like environment  $H_e$  via a Heisenberg ( $\Delta = -0.075$ ) Hamiltonian  $H_{ce}$ . The environment spins are initially prepared in the ground state. The different curves correspond to different  $K$ , but  $\sqrt{K}\Omega = 0.15\sqrt{N-1}$  is fixed. Solid (black) line:  $K = 2$ ; Dashed (red) line:  $K = 4$ ; Dotted (green) line:  $K = 6$ , and dash-dotted (blue) line:  $K = N - 1$ .

is more difficult for the central system to change from a pure quantum state to a classical, mixed state, which is of course consistent with the fact that the quantum effects become more prominent as the temperature decreases. In particular, from Fig. 6 it is clear that  $\sqrt{K}\Omega$  is not enough to characterize the qualitative behavior of the Loschmidt echo for the cases shown.

The difference between the cases of an environment at low-temperature<sup>14,15</sup> and a high-temperature (chaotic) environment considered in the present paper is most important for the systems with very large connectivity. In the latter case, the ground state of the environment is a quantum spin-glass which is a very effective source of decoherence<sup>14,15</sup>. At the same time, for the case of infinite temperature of the bath considered in this paper, this case is not very special when compared to the case of short-range interactions within the environment (see Fig. 5). It would be of interest to see if, as the temperature decreases, the decoherence process changes as the environment goes into the spin-glass state (at  $T \propto \sqrt{K}\Omega$ ), a problem that we leave for future research.

## VI. APPENDIX

Consider a generic quantum model described by the Hamiltonian  $H = H_c + H_{ce} + H_e$ , where  $H_c$  and  $H_e$  describe the central system and the bath respectively ( $[H_c, H_e] = 0$ ), and  $H_{ce}$  describes the coupling between them. If  $[H_c, H_{ce}] = 0$ , then the time evolution operator

of the whole system  $e^{-iHt}$  can be represented as

$$e^{-iHt} = e^{-iH_c t} e^{-i(H_{ce} + H_e)t}. \quad (6)$$

Denote the eigenstates and corresponding eigenvalues of the central system by  $\{|k\rangle\}$  and  $\{E_k\}$ , that is,  $H_c |k\rangle = E_k |k\rangle$ . The initial state ( $|\varphi(t_0)\rangle$ ) of the central system can be represented as  $|\varphi(t_0)\rangle = \sum_k a_k |k\rangle$ . For an isolated central system ( $H_{ce} = 0$ ), the time evolution of the density matrix of the central system is given by

$$\rho_0(t) = \sum_{k,l} e^{-i(E_k - E_l)t} a_k a_l^* |k\rangle \langle l|. \quad (7)$$

If the central system is coupled to the bath ( $|\phi(t_0)\rangle$ ), the initial state of the whole system can be represent as

$$|\Psi(t_0)\rangle = \sum_k a_k |k\rangle |\phi(t_0)\rangle, \quad (8)$$

and the state at later time  $t$  is

$$\begin{aligned} |\Psi(t)\rangle &= e^{-iHt} |\Psi(t_0)\rangle \\ &= \sum_k e^{-iE_k t} a_k e^{-i(H_{ce} + H_e)t} |k\rangle |\phi(t_0)\rangle. \end{aligned} \quad (9)$$

As  $[H_c, H_{ce}] = 0$ , we have  $H_{ce} |k\rangle |\phi(t_0)\rangle = |k\rangle M_k |\phi(t_0)\rangle$ , therefore

$$\begin{aligned} &e^{-i(H_{ce} + H_e)t} |k\rangle |\phi(t_0)\rangle \\ &= \sum_m \frac{(-it)^m (H_{ce} + H_e)^m}{m!} |k\rangle |\phi(t_0)\rangle \\ &= \sum_m |k\rangle \frac{(-it)^m (M_k + H_e)^m}{m!} |\phi(t_0)\rangle \\ &= |k\rangle e^{-i(M_k + H_e)t} |\phi(t_0)\rangle \\ &= |k\rangle |\phi_k(t)\rangle, \end{aligned} \quad (10)$$

where we introduced

$$|\phi_k(t)\rangle \equiv e^{-i(M_k + H_e)t} |\phi(t_0)\rangle. \quad (11)$$

Hence, the state at time  $t$  becomes

$$|\Psi(t)\rangle = \sum_k a_k e^{-iE_k t} |k\rangle |\phi_k(t)\rangle. \quad (12)$$

The density matrix  $\rho(t)$  of the whole system is

$$\begin{aligned} \rho(t) &= |\Psi(t)\rangle \langle \Psi(t)| \\ &= \sum_{k,l} e^{-i(E_k - E_l)t} a_k a_l^* |k\rangle |\phi_k(t)\rangle \langle l| \langle \phi_l(t)|, \end{aligned} \quad (13)$$

and the reduced density matrix  $\rho_c(t)$  of the central system is

$$\begin{aligned} \rho_c(t) &= \text{Tr}_e \rho(t) \\ &= \sum_{k,l} e^{-i(E_k - E_l)t} a_k a_l^* \langle \phi_l(t) | \phi_k(t) \rangle |k\rangle \langle l|. \end{aligned} \quad (14)$$



The Loschmidt echo  $L(t)$  of the central system can be calculated as

$$\begin{aligned}
L(t) &= \text{Tr}(\rho_c(t) \rho_0(t)) \\
&= \text{Tr} \left[ \sum_{k,l} e^{-i(E_k - E_l)t} a_k a_l^* \langle \phi_l(t) | \phi_k(t) \rangle |k\rangle \langle l| \right. \\
&\quad \times \sum_{m,n} e^{-i(E_m - E_n)t} a_m a_n^* |m\rangle \langle n| \left. \right] \\
&= \text{Tr} \left[ \sum_{k,l,n} e^{-i(E_k - E_n)t} a_k |a_l|^2 a_n^* \right. \\
&\quad \times \langle \phi_l(t) | \phi_k(t) \rangle_l |k\rangle \langle n| \left. \right] \\
&= \sum_{k,l} |a_k|^2 |a_l|^2 \langle \phi_l(t) | \phi_k(t) \rangle. \tag{15}
\end{aligned}$$

It is clear that if  $[H_c, H_{ce}] = 0$ , the decoherence process is determined by the initial state of the central system  $\{a_k\}$  and the time evolution of the  $\{|\phi_k(t)\rangle\}$ . As shown in Eq. (11), the  $\{|\phi_k(t)\rangle\}$  are determined by the initial state of the bath ( $|\phi(t_0)\rangle$ ), the eigenstates  $\{|k\rangle\}$  of the central system, and the Hamiltonian  $H_{ce}$  and  $H_e$ . The eigenvalues  $\{E_k\}$  have no effect of the decoherence process. Thus, multiplying  $H_c$  by a constant does not change the  $L(t)$  and the diagonal elements of the reduced den-

sity matrix  $\rho_c(t)$ . The time evolution of the absolute value of the off-diagonal elements

$$|\rho_c(t)_{kl}| = |a_k a_l^*| \langle \phi_l(t) | \phi_k(t) \rangle, \tag{16}$$

is independent of  $H_c$ .

Finally, we consider the case that not only  $[H_c, H_{ce}] = 0$  but also  $[H_{ce}, H_e] = 0$ . Then, Eq. (11) becomes

$$|\phi_k(t)\rangle = e^{-i(M_k + H_e)t} |\phi(t_0)\rangle = e^{-iM_k t} e^{-iH_e t} |\phi(t_0)\rangle, \tag{17}$$

therefore we have

$$\begin{aligned}
\langle \phi_l(t) | \phi_k(t) \rangle &= \langle \phi(t_0) | e^{iH_e t} e^{iM_l t} e^{-iM_k t} e^{-iH_e t} | \phi(t_0) \rangle \\
&= \langle \phi(t_0) | e^{-i(M_k - M_l)t} | \phi(t_0) \rangle, \tag{18}
\end{aligned}$$

implying that  $|\rho_c(t)_{kl}|$  and  $L(t)$  do not depend on  $H_e$ .

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